$\boxed{14.1}$ Partial Derivatives for $z = f(x,y)$ — Other notation

$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h} \qquad \frac{\partial z}{\partial x}$$

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h} \qquad \frac{\partial z}{\partial y}$$

at the point (a,b)

$$f_x(a,b) = \left.\frac{\partial z}{\partial x}\right|_{(a,b)} \qquad f_y(a,b) = \left.\frac{\partial z}{\partial y}\right|_{(a,b)}$$

Note: We have used the notation f_x and f_y to represent $f_x(x,y)$ and $f_y(x,y)$

$\boxed{14.4}$ the directional derivative of $z = f(x,y)$ at (a,b) in the direction of the unit vector $\vec{u} = u_1\vec{i} + u_2\vec{j}$ is

$$f_{\vec{u}}(a,b) = \lim_{h \to 0} \frac{f(a+hu_1, b+hu_2) - f(a,b)}{h}$$

$$= f_x(a,b)u_1 + f_y(a,b)u_2$$

If $\quad \text{grad} f(a,b) = \nabla f(a,b) = f_x(a,b)\vec{i} + f_y(a,b)\vec{j}$

then $\quad f_{\vec{u}}(a,b) = \nabla f(a,b) \cdot \vec{u} \overset{\text{Sec 13.3}}{=} \|\nabla f\| \|\vec{u}\| \cos\theta$

$\Rightarrow$ of all directions to travel from (a,b), ∇f gives the direction of max change for z.