Math 32
Test 2

Name Key

Directions: Show your work neatly and clearly. Answers without adequate justification will not receive full credit.

Solve each equation.

1. Solve: $x = 1 + \frac{1}{x}$ multiply by x but not correct -6

$x^2 = x + 1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$= \boxed{\frac{1 \pm \sqrt{5}}{2}}$

2. Find all zeros: $f(x) = x^4 - 16$

$x^4 - 16 = 0$

$(x^2-4)(x^2+4) = 0$

$(x+2)(x-2)(x^2+4) = 0$

$\boxed{x = -2, 2, \pm 2i}$

-5

3. List all possible rational zeros for $f(x) = 6x^3 - 21x^2 + 10x - 2$. Don't find the actual zeros, just list the possibilities.

$\text{Possibilities} = \pm \frac{1, 2}{1, 2, 3, 6} = \pm\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 2, \frac{2}{3}\right)$

-2

Scale	
90 A	5
78 B	8
60 C	6
50 D	0
↓ F	5
	24

4. Find all zeros for $f(x) = x^4 + x^3 - x^2 + x - 2$

1 | 1 1 −1 1 −2
↓ 1 2 1 2
1 2 1 2 | 0 yes

−1 | 1 2 1 2
↓ −1 −1 0
1 1 0 2 NO

1 | 1 2 1 2
↓ 1 3 4
1 3 4 | 6 NO

−2 | 1 2 1 2
↓ −2 0 −2
1 0 1 | 0 yes

$x^4 + x^3 - x^2 + x - 2 = (x-1)(x+2)(x^2+1)$

$\Rightarrow \boxed{x = 1, -2, \pm i}$ zeros

-5

5. Name the horizontal asymptote for the graph of each equation.

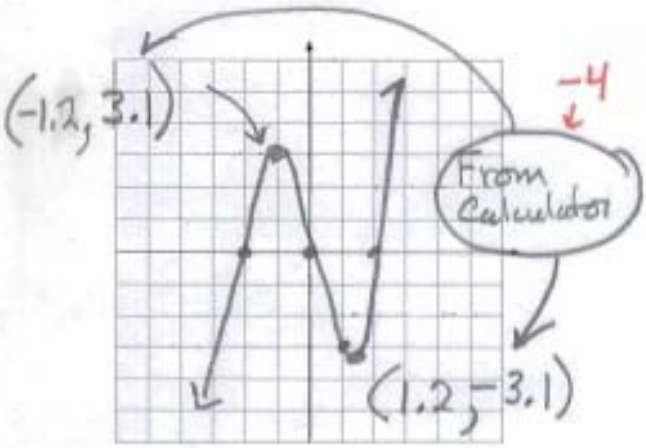
a. $y = \frac{3x-4}{5x+1}$

$y = \frac{3}{5}$ graph, but wrong −3

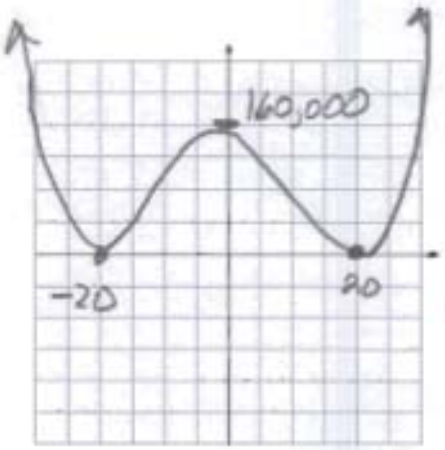
b. $y = \frac{x}{x^2-9}$

$y = 0$

6. Graph $f(x) = x^3 - 4x$
Label all intercepts accurately.
Label the maximum and minimum points accurately.



7. Graph $f(x) = (x-20)^2(x+20)^2$.
Label all intercepts accurately.



wrong scale −5

x-intercepts:

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

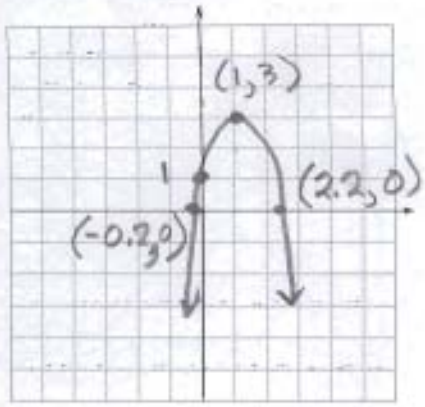
$x = 0, -2, 2$

Other points

$x = 1, \; y = 1^3 - 4 \cdot 1 = -3$

$x = -1, \; y = (-1)^3 - 4(-1) = 3$

8. Graph $f(x) = -2x^2 + 4x + 1$
Label the intercepts and the vertex accurately.



x-intercepts:

$$-2x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-2)(1)}}{2(-2)}$$

$$= \frac{-4 \pm \sqrt{24}}{-4}$$

$$= \frac{-4 \pm 2\sqrt{6}}{-4}$$

$$= \boxed{\frac{2 \pm \sqrt{6}}{2}} \begin{cases} \to 2.22 \\ \to -0.22 \end{cases}$$

Vertex

$$x = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$$

$$y = -2 + 4 + 1 = 3$$

9. Graph $y = x^3 + 5x^2 + 7x - 3$
Label the intercepts accurately.

x-intercepts

1	1	-5	7	-3
	↓	1	-4	3
	1	-4	3	0 yes

$$x^3 + 5x^2 + 7x - 3 = (x-1)(x^2 - 4x + 3)$$

$$= (x-1)(x-3)(x-1)$$

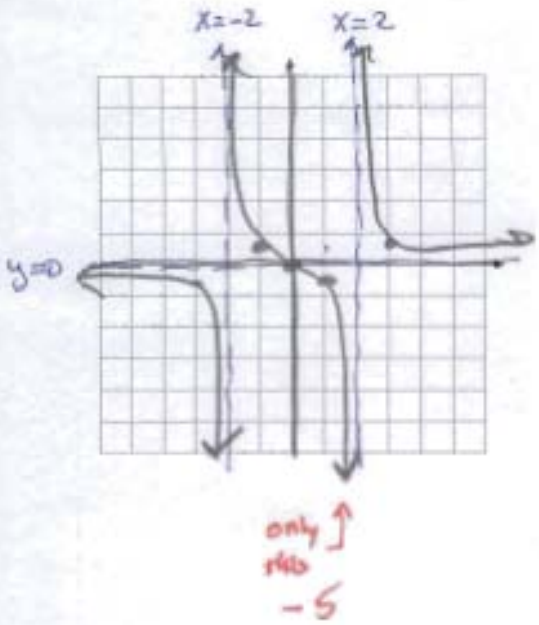
$\Rightarrow$ x-intercepts at 1, 3

y-intercept: $x = 0, y = -3$

other point

$x = 2, y = 8 - 20 + 14 - 3 = -1$

10. Graph $y = \frac{x}{x^2 - 4}$ Label all intercepts and asymptotes.



Horizontal asymptote: $y = 0$ (x-axis)

Vertical asymptote: $x = \pm 2$

intercepts at $(0,0)$

Points

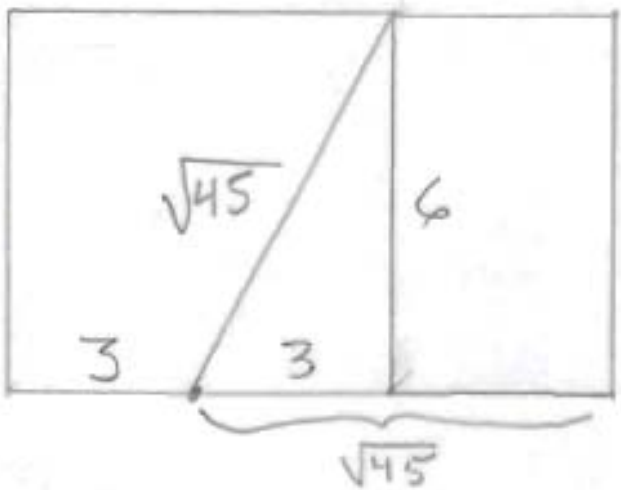
$x = 1, \; y = \frac{1}{-3} = -\frac{1}{3}$

$x = 3, \; y = \frac{3}{9-4} = \frac{3}{5}$

$x = -1, \; y = \frac{-1}{-3} = \frac{1}{3}$

$x = -3, \; y = \frac{-3}{9-4} = -\frac{3}{5}$

11. For 5 points extra credit, construct a golden rectangle from a square of side 6 and show that the ratio of length to width is the golden ratio $\frac{1+\sqrt{5}}{2}$.



$$\frac{L}{w} = \frac{3+\sqrt{45}}{6}$$

$$= \frac{3+3\sqrt{5}}{6}$$

$$= \frac{1+\sqrt{5}}{2}$$