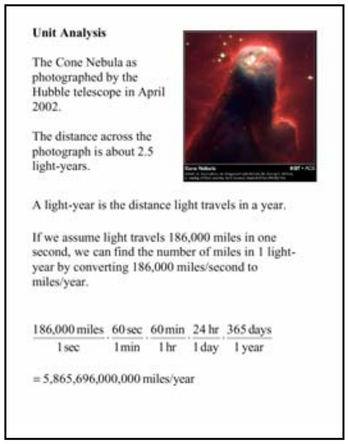
Astronomy, Postage Stamps, Conics, and Eccentricity

Unit Analysis

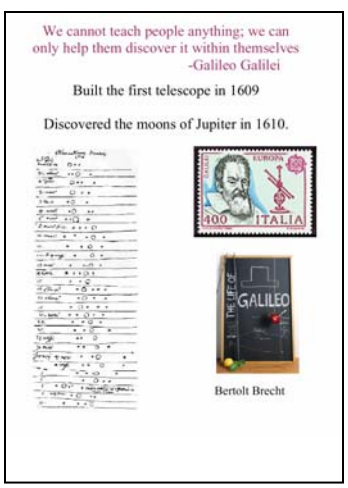
Transparency 1 shows the Cone Nebula as it appeared in USA Today. We can discuss how technology has improved what we see in the universe, and that people are debating today what it is that we are seeing. This overhead uses the Cone Nebula as a lead-in to unit analysis, or scientific notation.



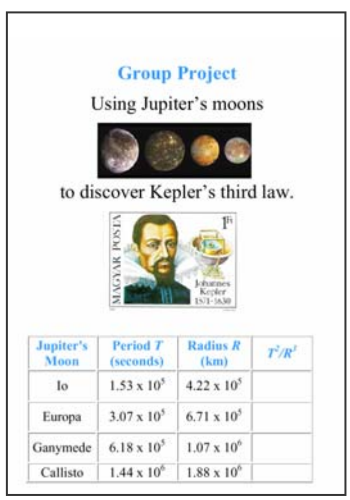
Transparency 1

Scientific Notation

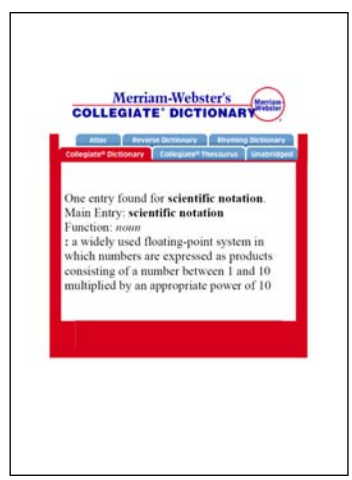
Put up Transparency 1 that was used previously, and remind them that the photograph was taken with the Hubble telescope. As technology has improved, so has our ability to see more of the details in space. What was the first technology that allowed us to improve our view of the sky? Put up Transparency 2. The table is from Galileo's notebook and shows the positions of four of Jupiter's moons. This leads into Transparency 3, which is a group project that will require students to work with numbers in scientific notation. Show the website that demonstrates Kepler's third law.



Transparency 2



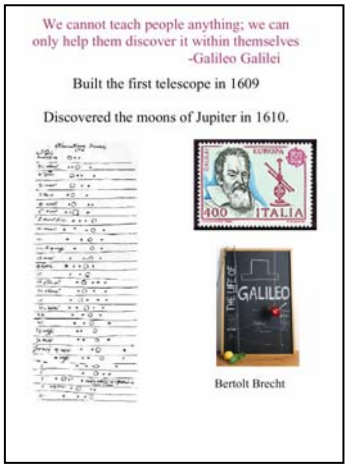
Transparency 3



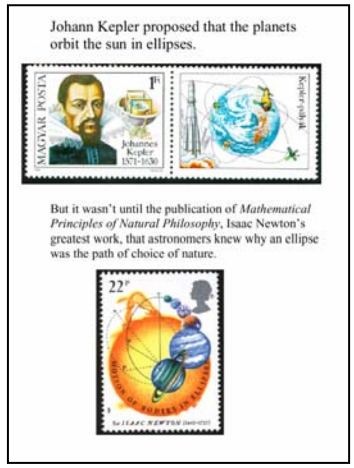
Transparency 21

Conic Sections

We introduce the conic sections by starting with just the ellipse. To start, put up Transparency 2 again to remind them that the first telescope was built in 1609. This leads into Transparency 4 which shows that in 1609 Kepler proposed that the planets orbit the sun in elliptical orbits. Kepler used empirical data for his conjecture. It took Newton to actually prove that a planetary orbit had to be a conic section.



Transparency 2



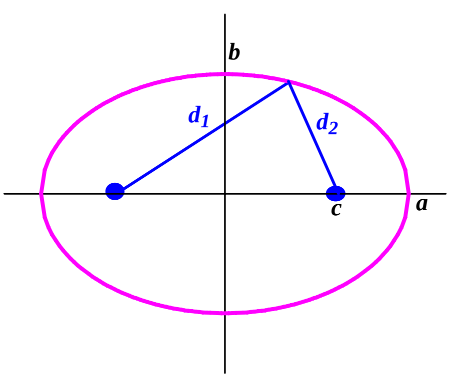
Transparency 4

Next, show the website demonstrating Kepler's first and second laws, so they can see an ellipse. Then draw an ellipse using the snaps and string on the overhead projector. To remind you of how we did that in the talk, here are the steps:

Deriving the Rectangular Equation for an Ellipse There are no transparencies for this page. This is just to remind you of the way we developed the initial facts about an ellipse using the string and snaps to draw an ellipse.

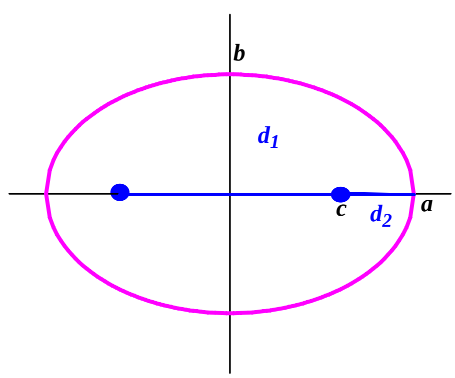
Step 1 After drawing an ellipse with the string, draw d_1 and d_2 out to an arbitrary point and write

$$d_1 + d_2 = \text{constant}$$



Step 2 Then move the arbitrary point to a to show that

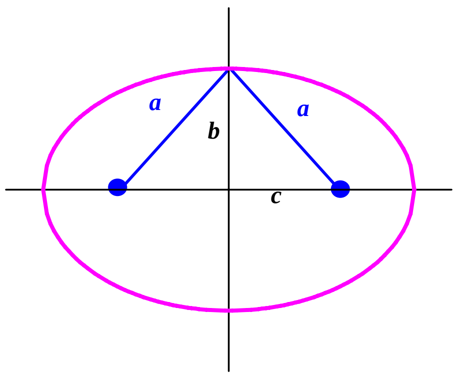
$$d_1 + d_2 = 2a$$



Conic Sections (Continued)

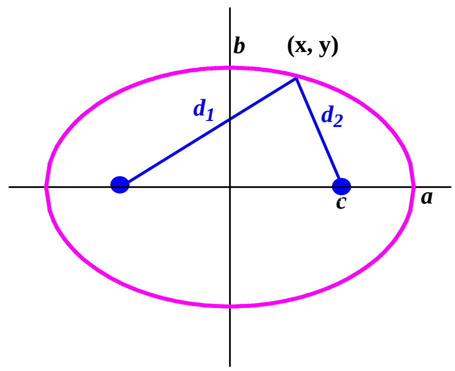
Step 3 Next, move the arbitrary point to the top of the ellipse to show that

$$a^2 = b^2 + c^2$$



Step 4 Now move back to the original arbitrary point and use the distance formula, along with the results of steps 1, 2, and 3 to derive the equation of the ellipse

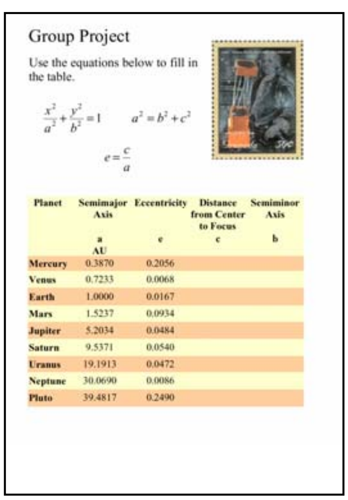
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



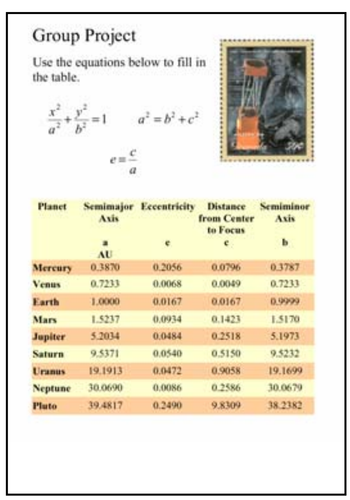
At this point, I would also introduce the eccentricity so that we can work with the data that is given by astronomers.

Eccentricity: $e = c / a$

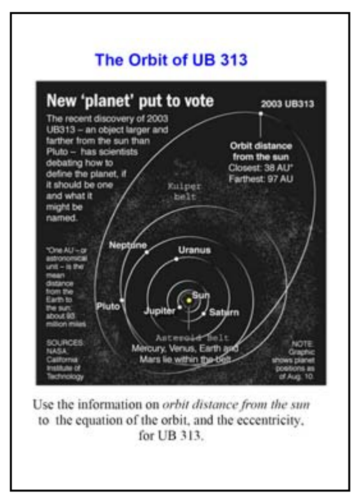
Now we are ready to put these equations to use. Transparencies 5 and 6 lead into the interesting fact that, even though the orbits of the planets are ellipses, they are very close to circles.



Transparency 5



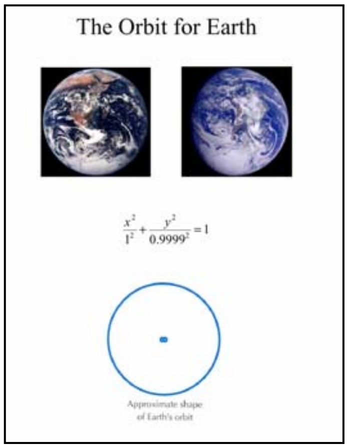
Transparency 6



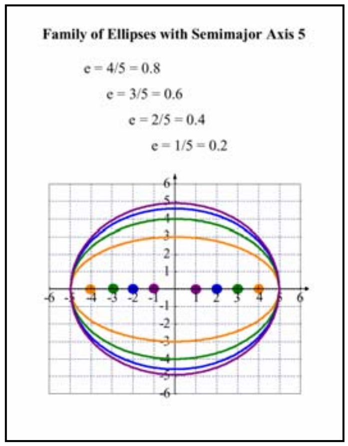
Transparency 20

Conic Sections (Continued)

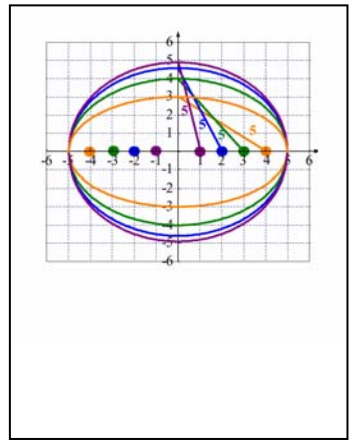
Transparency 7 shows two nice views of earth, along with the equation and the graph of its orbit. Transparencies 8 and 9 give us a chance to discuss eccentricity and reinforce the geometric relationship between *a*, *b,* and *c*.



Transparency 7

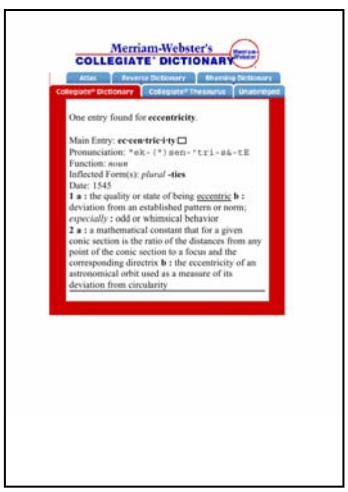


Transparency 8

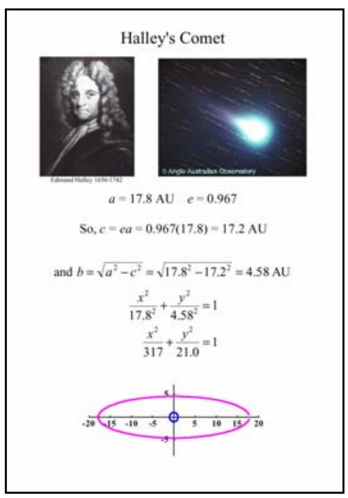


Transparency 9

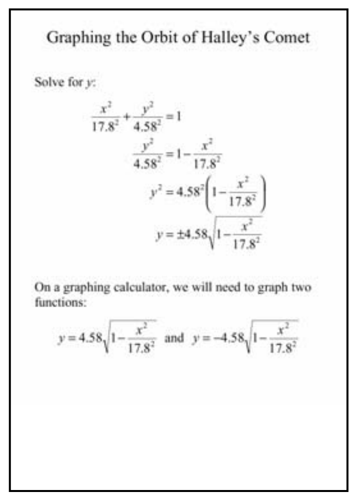
The definition for eccentricity in Transparency 10 is from the Internet (and the second part of the definition is what we use to introduce conic sections in polar coordinates). To show an orbit with an eccentricity close to 1, we can use Halley's Comet in Transparency 11. To graph the orbit of Halley's Comet on a graphing calculator we need to solve for *y*, as shown in Transparency 12.



Transparency 10



Transparency 11

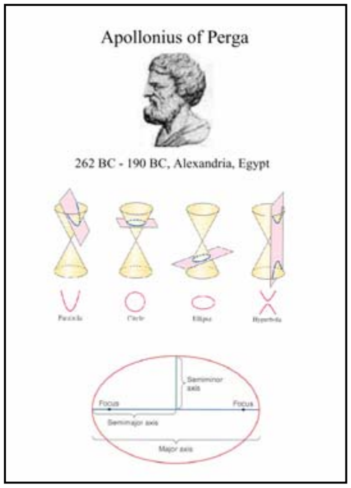


Transparency 12

As you know, it became apparent to me that I don't cover eccentricity in as much detail as I should in intermediate or college algebra. Now that I see how the astronomers give information about orbits, I am going to do more with it in my classes. These overheads may be a good way to start.

Conic Sections (Continued)

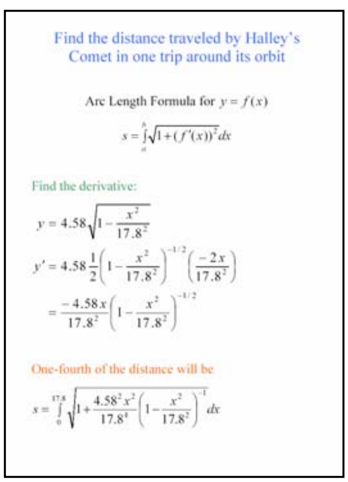
Now we are ready to introduce the rest of the conics. Transparency 13 does this very nicely. The idea is to show that conic sections have been studied many years before there were any applications discovered; the value of mathematics is not necessarily in its applications. It is interesting to note that, at the same time Apollonius was studying conic sections, the first humans were rowing their way to Hawaii.



Transparency 13

Arc Length

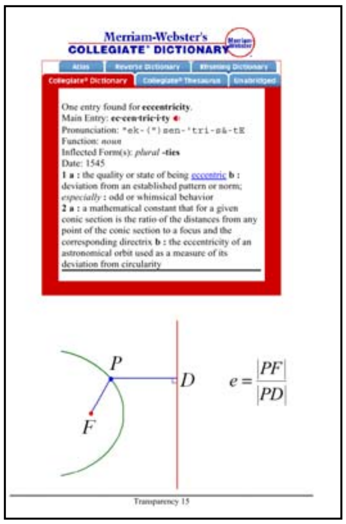
In calculus, after we have introduced the formulas for arc length, we can bring back Halley’s Comet and use the graphing calculator to approximate the distance traveled by Halley’s Comet in one trip around its orbit. This is shown in Transparency 14. Remember, we approximated the integral using the graphing calculator.



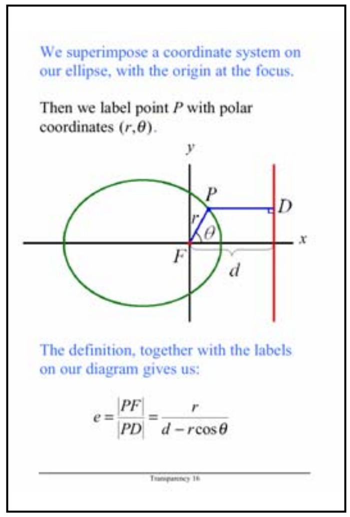
Transparency 14

Conic Sections in Polar Coordinates

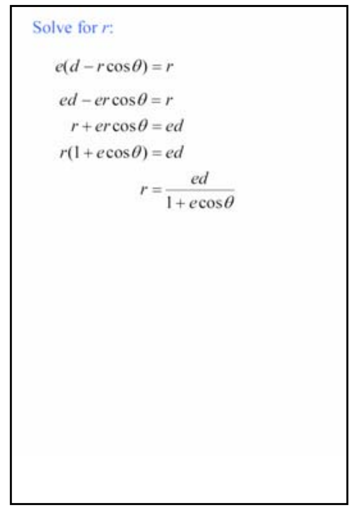
Use the online dictionary definition for eccentricity as a lead-in to polar equations for conics. The diagram in Transparency 15 is drawn from this definition. Transparency 16 shows an ellipse drawn from the online definition, with a coordinate system superimposed on it so that the origin of the coordinate system is at the focus. Point *P* has polar coordinates (r, θ). Drop a line segment from *P* to the *x*-axis at a right angle. This forms a triangle with base $r\cos\theta$. Transparency 17 shows how we manipulate this formula to find an equation for the conic with *r* given in terms of θ.



Transparency 15

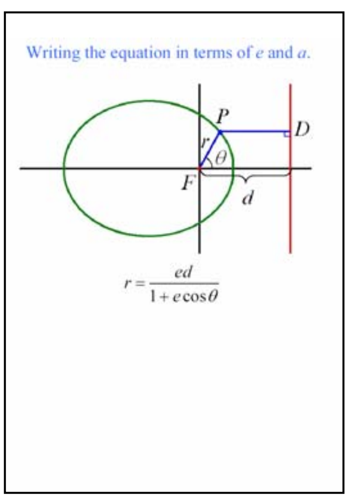


Transparency 16

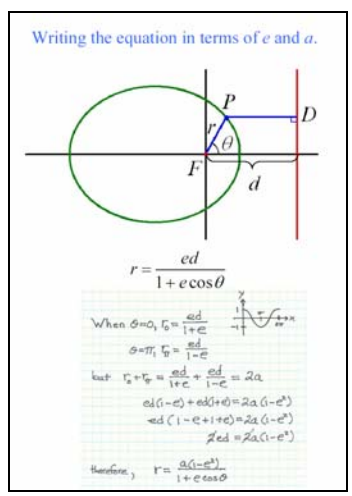


Transparency 17

Transparency 18 can be used to derive the equation of the conic in terms of only *a* and *e*, the form used by astronomers. Transparency 19 shows how we did this during the talk.



Transparency 18



Transparency 19

I hope you find these transparencies useful

Web Links for Conic Sections talk:

The first two links are to the demonstrations of Kepler's laws that were shown during the talk. They come from Dr. Les Tomley at San Jose State University.

Kepler's first and second laws:

http://www.physics.sjsu.edu/tomley/Kepler12.html

Kepler's third law:

http://www.physics.sjsu.edu/tomley/Kepler3.html

Solar System animation:

http://astro.u-strasbg.fr/~koppen/orbitviewer/halley.html

Postage Stamps of Mathematicians

http://jeff560.tripod.com/

Powers of 10 animation:

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html