

58. Unit vector from $(2, 1)$ to $(1, 3)$ is $\vec{u} = -\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$

Unit vector from $(2, 1)$ to $(5, 5)$ is $\vec{v} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

Gradient at $(2, 1)$ is $\nabla f = f_x\vec{i} + f_y\vec{j}$

Directional Derivatives:

Given in book

$$f_{\vec{u}}(1, 2) = \nabla f \cdot \vec{u} = -\frac{1}{\sqrt{5}}f_x + \frac{2}{\sqrt{5}}f_y = -\frac{2}{\sqrt{5}}$$

Multiply both sides
by $\sqrt{5}$ }
$$\boxed{-f_x + 2f_y = -2} \quad (\text{Eq 1})$$

$$f_{\vec{v}}(1, 2) = \nabla f \cdot \vec{v} = \frac{3}{5}f_x + \frac{4}{5}f_y = 1$$

Multiply both sides
by 5 }
$$\boxed{3f_x + 4f_y = 5} \quad (\text{Eq 2})$$

Solve the system of equations given
by (Eq 1) and (Eq 2) to get

$$f_x(1, 2) = \underline{\underline{1.8}} \quad \text{and} \quad f_y(1, 2) = -\frac{1}{10} = \underline{\underline{-0.1}}$$