

# The Human Cannonball, Ferris Wheels, and Trigonometry

Pat McKeague  
1339 Marsh Street  
San Luis Obispo, CA 93401  
(805) 541-4593  
pat@mckeague.com



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## Notes for Transparencies

### The Human Cannonball

**Day 1 (Algebra Review):** Put up the web page for the Bullet. Then use the information given there to find the equation of the path of the Bullet.

**Day 2 (Right Triangle Trigonometry):** Using the graph and equation of the path of the Bullet, find the angle between the cannon and horizontal. Plot the point  $(x, y)$  and draw the right triangle with sides  $x$  and  $y$ , so that the hypotenuse of the right triangle approximates the cannon, and angle  $\theta$  approximates the angle between the cannon and the horizontal. Then use the table function on the graphing calculator to find  $\theta$  exactly.

**Day 3 (Vectors):** Using the Bullet for a model, find the initial velocity of the Bullet as he leaves the cannon. This requires some elementary concepts from physics.

**Day 4 (Parametric Equations):** Find the position of the Bullet as a function of time using parametric equations. Again, a little physics is needed here.

**Day 5 (Parametric Equations):** Graph the different paths taken by a human cannonball as we vary the angle between the horizontal and the cannon, keeping the initial velocity at 50 miles per hour. This can be accomplished easily using the graphing calculator.

### Ferris Wheels

**Day 1 (Angular Velocity):** Put up the picture of the first Ferris wheel. Then put up the Vienna wheel and explain that it appears in the movie *The Third Man* with Orson Welles. At the end of the movie, Mr. Welles rides the wheel through one complete revolution. The trip takes 3.8 minutes. Calculate the linear and angular velocity of Mr. Welles. (The diameter of this wheel is 197 feet. The bottom of the wheel is 12 feet off the ground.)

**Day 2 (Parametric Equations):** Put up the picture of the Vienna wheel. Superimpose a coordinate system over it, and then find the path of a rider as a function of  $t$ , and as a function of time.

**Day 3 (Parametric Equations):** Using a graphing calculator, "shoot" one of the Zacchinis over the Vienna wheel. To shoot the Bullet out of the cannon at 100 miles per hour, change the initial velocity in the parametric equations to 147.

THE  
*Great*  
**ZACCHINI'S**



History in Pictures

*by Pat Ringling Buck*



# Washington County

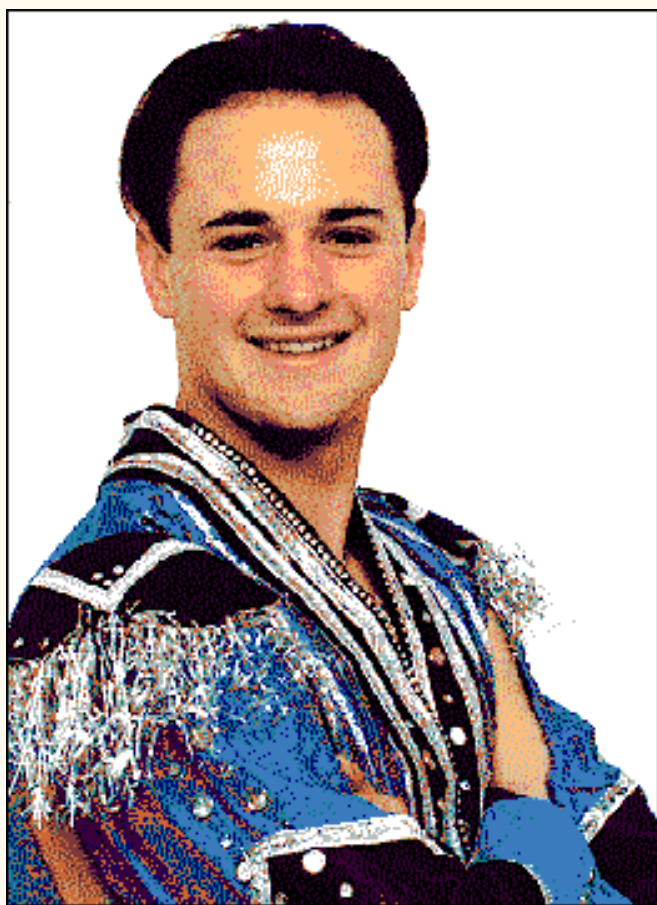
## Oregon

### 1997 Fair & Rodeo:

Last Updated: July 23, 1997

Fair Complex  
Cal Wade, Executive Director

down



## Human Cannonball

Tues-Fri, 3:30 & 8:00 PM 30  
Sat, 12:30, 3:00 & 8:00 PM  
Sun, 12:30, 3:30 & 6:30 PM

### Cannonball Landing Site

As the countdown begins, David Smith, Jr. - THE BULLET, dons his safety helmet, slides down the slender barrel of his cannon positioning himself to become a human projectile. The crowd joins in the countdown and the announcer asks the audience to cover their ears as protection from the noise of the blast. Three! Two! One! FIRE! David Smith, Jr. - THE BULLET - blasts into the air, flies an average distance of 160 feet and a height of 60-70 feet and lands in his pre-positioned safety net. This is a death-defying feat guaranteed to send shivers down

your spine!

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**Fair Food:** Sit in one of our many picnic areas and enjoy such treats as barbecued ribs, burritos, pizza, mocha & bagels, Caesar salad, Australian meat sandwiches, pasta dishes, and your favorite fair sweets.

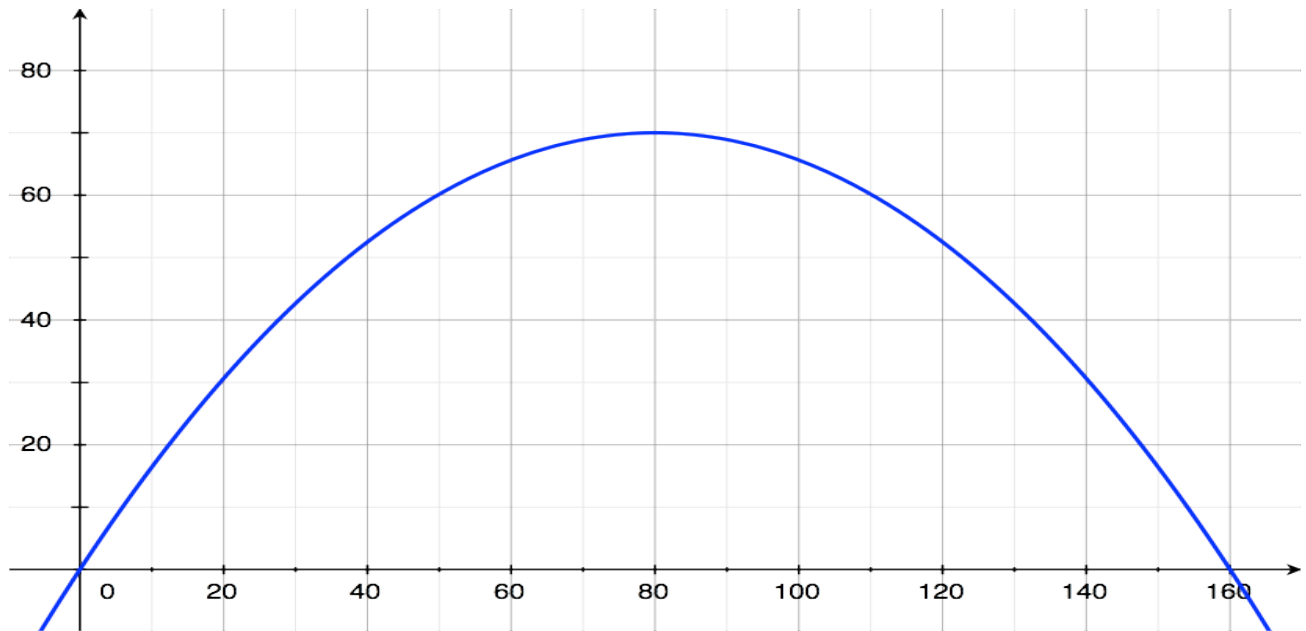
**Fair Hours and Admission:** Fair hours are 10:30 AM to 11:00 PM, Tuesday through Saturday, and 10:30 AM to 9:00 PM on Sunday. Admission is \$6 for adults, \$3.00 for youth ages 6 to 15 and children under 6 are free.

*Discount coupons* of \$1 are available at local BI-MART stores. Seniors receive free fair admission on Senior Discount Day, Wednesday, July 30.

## The Bullet

Maximum Height: 70 feet    Maximum distance: 160 feet





The curve is a parabola:

$$y = a(x - h)^2 + k \quad \text{for } 0 \leq x \leq 160$$

$$\text{Vertex: } (80, 70) \Rightarrow y = a(x - 80)^2 + 70$$

Starting point:  $(0, 0)$

Ending point:  $(160, 0)$

$$0 = a(0 - 80)^2 + 70$$

$$0 = a(160 - 80)^2 + 70$$

Either way

$$0 = 6400a + 70 \Rightarrow a = -\frac{70}{6400} = -\frac{7}{640}$$

Final Equation

$$y = -\frac{7}{640}(x - 80)^2 + 70 \quad \text{for } 0 \leq x \leq 160$$

As his children grew older, Ildebrando began to retire from active participation in the circus and return to his first love, painting. He had used his art to do wonderful advertising for the circus, painting on canvas the scenes from the pantomimes and other offerings of the troupe and displaying the work outside the tent like a bannerline. But portraits and religious paintings were what he wanted to paint and he began to concentrate on these subjects. Papa Zacchini still had a contribution to make to the circus, however, and it would be his crowning achievement.



*Zacchini Circus clowns:  
Edmondo, center, Hugo, second from right, Bruno, right*



*Zacchini Circus tours Egypt with the cannon act*

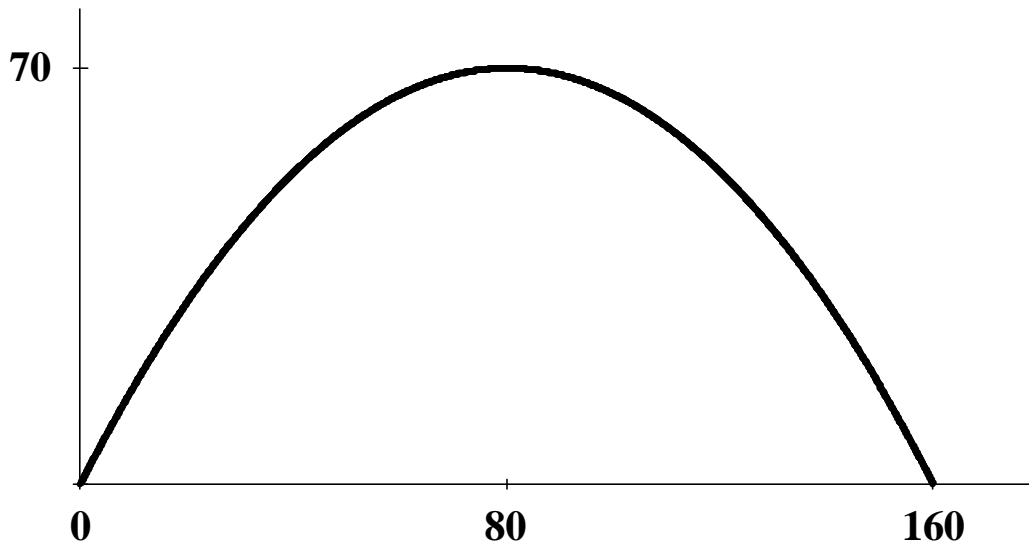
During the First World War, Ildebrando had been inspired to work on a device that could catapult a man behind enemy lines. When the government showed no interest in the plan, the Zacchini patriarch decided to adapt his invention for a circus act and continued to work on it. Finally he enlisted the assistance of his three oldest sons - Edmondo, Hugo and Bruno - to develop the first Zacchini Human Cannon.

In 1922, with a cannon constructed by an engineering firm in Malta, the Zacchini cannonball act had its world premiere in Cairo, Egypt and Hugo Zacchini shot into the spotlight as the first human cannonball. (Because of his bad leg, Edmondo was never able to go out of the cannon but Bruno served as a "guinea pig" for his father while the act was being developed.)

## The Bullet

Maximum height: 70 feet      Maximum distance: 160 feet

Equation:  $y = -\frac{7}{640}(x - 80)^2 + 70$  for  $0 \leq x \leq 160$



What is the angle between the cannon and the horizontal?

*Y Variables Setup*

$$Y1 = -\frac{7}{640}(X - 80)^2 + 70$$

$$Y2 = Y1/X$$

$$Y3 = \tan^{-1}(Y2)$$

*Table Setup*

Start = 0.01

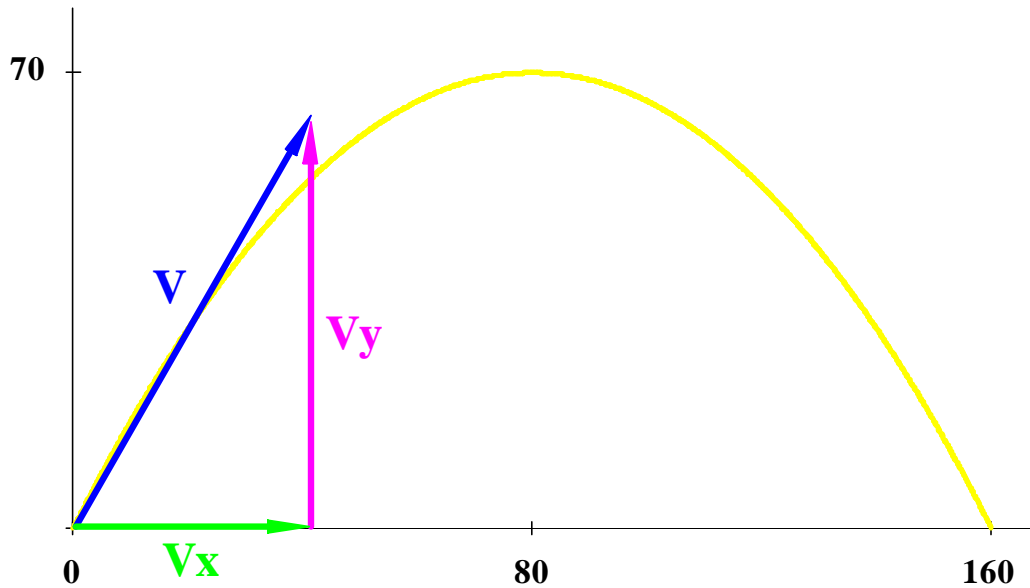
Increment = 0.01

Variables: Auto

## The Bullet

Maximum height: 70 feet      Maximum distance: 160 feet

Cannon angle:  $60.3^\circ$



What is the initial velocity of The Bullet?

(First find the magnitude of  $V_y$ , its initial vertical speed.)

How long does it take for  
any object to fall 70 feet?

$$\begin{aligned}y &= 16t^2 \\70 &= 16t^2 \\t &= \sqrt{\frac{70}{16}} \\t &\approx 2.092 \text{ seconds}\end{aligned}$$

What is its speed when  
it hits the ground?

$$\begin{aligned}|V_y| &= 32t \\&\approx 32(2.092) \\&= 66.94 \text{ ft/sec}\end{aligned}$$

$$\begin{aligned}\sin 60.3^\circ &= \frac{66.94}{|V|} \quad \Rightarrow \quad |V| = \frac{66.94}{\sin 60.3^\circ} = 77.0 \text{ ft/sec} \\&= 52.5 \text{ mi/hr}\end{aligned}$$



*1944, Silvana is first female cannonball*

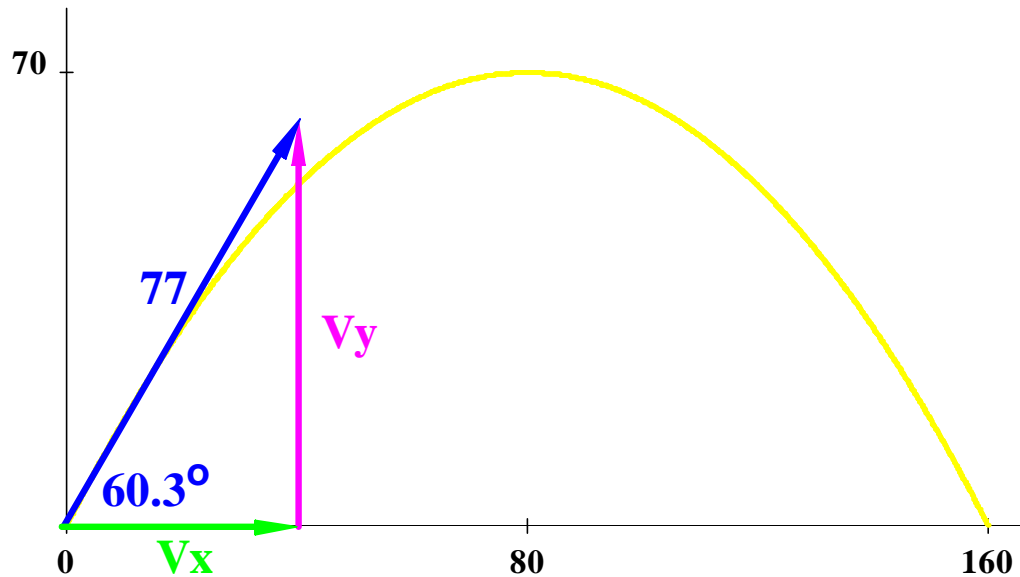
## The Bullet

Maximum height: 70 feet

Maximum distance: 160 feet

Cannon angle:  $60.3^\circ$

Initial velocity:  $52.5 \text{ mi/hr} = 77 \text{ ft/sec}$



Can we write his position as a function of time?

$$\sin 60.3^\circ = \frac{|V_y|}{77} \Rightarrow |V_y| = 77 \sin 60.3^\circ$$

$$\cos 60.3^\circ = \frac{|V_x|}{77} \Rightarrow |V_x| = 77 \cos 60.3^\circ$$

$$d = r \cdot t$$

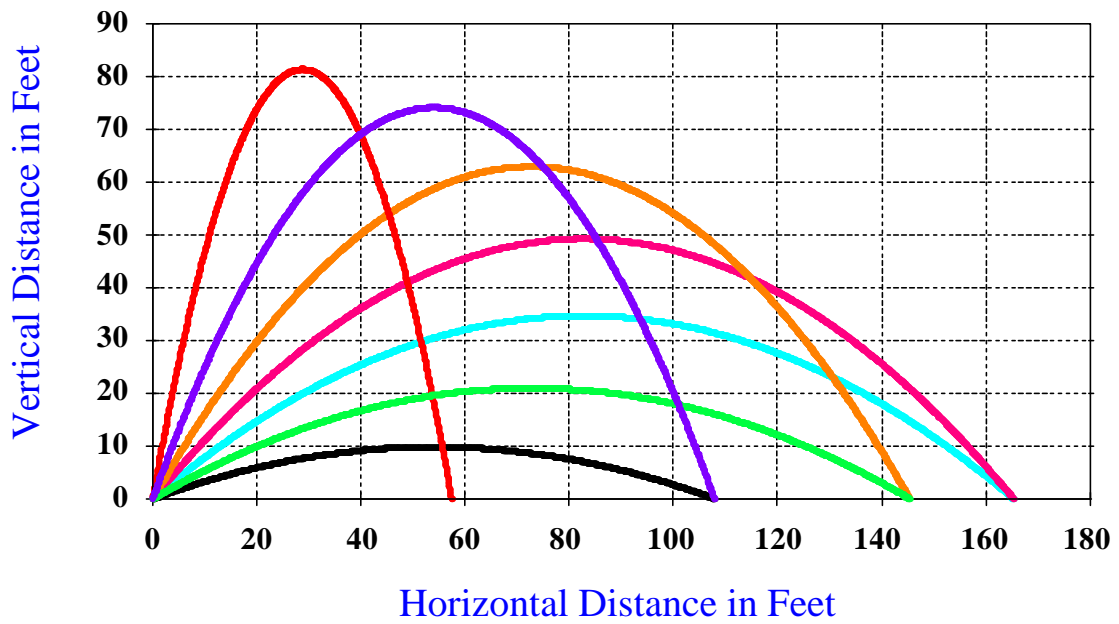
Horizontal distance:  $x = (77 \cos 60.3^\circ)t$

Vertical distance:  $y = \text{distance up} - \text{distance down}$   
 $= (77 \sin 60.3^\circ)t - 16t^2$

# Path of the Human Cannonball

Initial Velocity: 50 miles per hour

Angle:  $20^\circ$ ,  $30^\circ$ ,  $40^\circ$ ,  $50^\circ$ ,  $60^\circ$ ,  $70^\circ$ ,  $80^\circ$

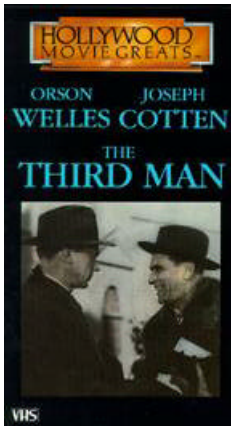


# The Ferris Wheel June 1893, Chicago



Diameter: 250 Feet  
Top of wheel is 264 feet above the ground  
36 carriages holding 60 people each  
1 revolution every 20 minutes

# The Vienna Giant Ferris Wheel



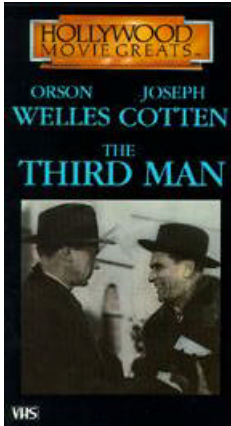
Velocities: From the movie *The Third Man*

$$\text{Angular: } \omega = \frac{2\pi \text{ radians}}{3.8 \text{ minutes}} = 1.65 \text{ radians/minute}$$

$$\text{Linear: } v = \frac{\pi(197 \text{ feet})}{3.8 \text{ minutes}} = 162.9 \text{ feet/minute}$$

$$\frac{162.9 \text{ ft}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5,280 \text{ ft}} = 1.85 \text{ miles/hour}$$

# The Vienna Giant Ferris Wheel



The path of *The Third Man*:

As a function of  $\theta$ :

$$x = 98.5 \cos \left( \theta - \frac{\pi}{2} \right)$$

$$y = 98.5 \sin \left( \theta - \frac{\pi}{2} \right)$$

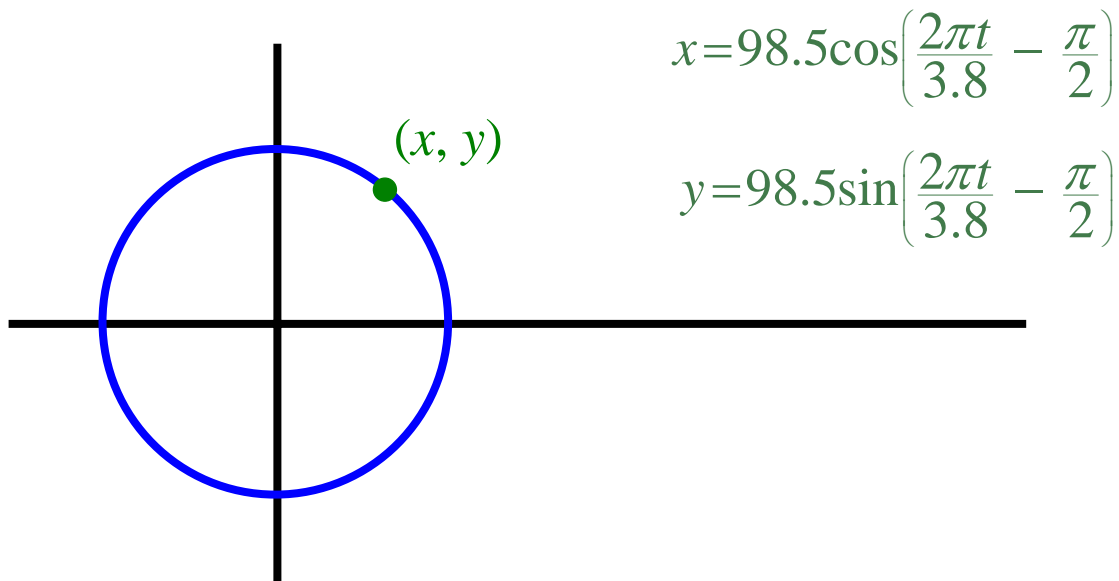
As a function of time:

$$\frac{\theta}{2\pi} = \frac{t}{3.8} \Rightarrow \theta = \frac{2\pi t}{3.8}$$

$$x = 98.5 \cos \left( \frac{2\pi t}{3.8} - \frac{\pi}{2} \right)$$

$$y = 98.5 \sin \left( \frac{2\pi t}{3.8} - \frac{\pi}{2} \right)$$

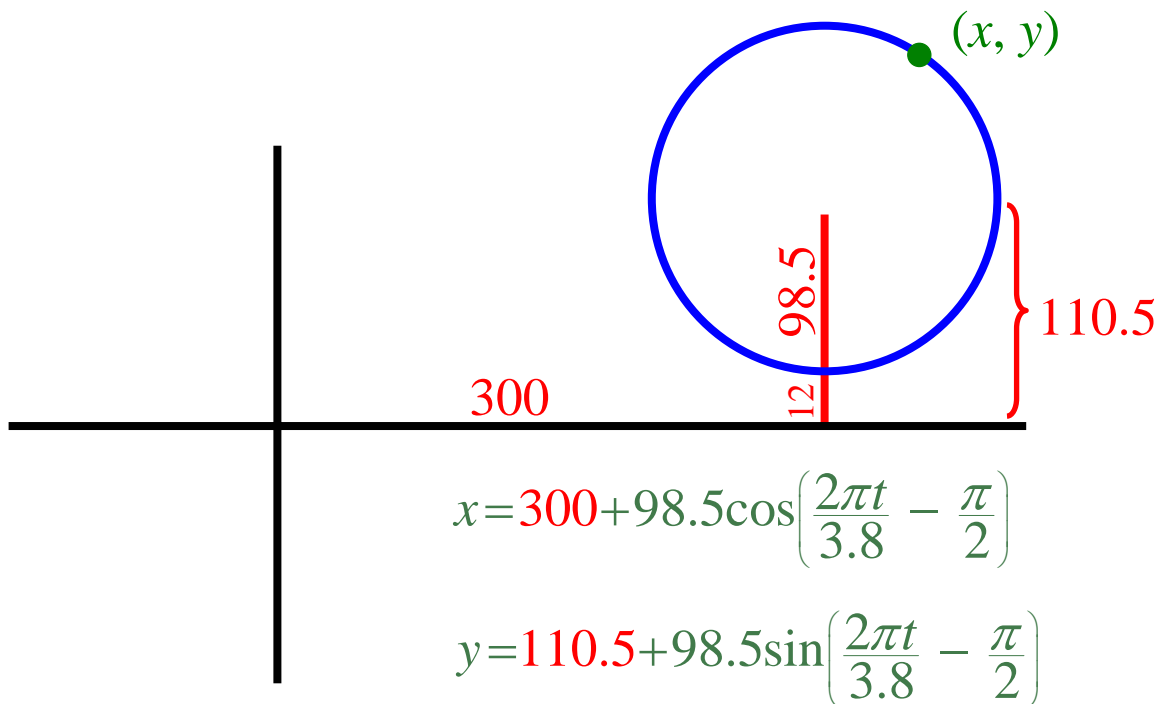
## Translating the Wheel Away from the Origin



$$x = 98.5 \cos\left(\frac{2\pi t}{3.8} - \frac{\pi}{2}\right)$$

$$y = 98.5 \sin\left(\frac{2\pi t}{3.8} - \frac{\pi}{2}\right)$$

Suppose the bottom carriage is 12 feet above the ground and 300 feet from the entrance to the park.



$$x = 300 + 98.5 \cos\left(\frac{2\pi t}{3.8} - \frac{\pi}{2}\right)$$

$$y = 110.5 + 98.5 \sin\left(\frac{2\pi t}{3.8} - \frac{\pi}{2}\right)$$